

Scientific Uses of the MANIAC

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This is the story of how Nick Metropolis came to build the MANIAC and about the scientific uses to which it was put in the early days of electronic computing. Among the illustrious scientists attracted to the MANIAC were Fermi, Teller, von Neumann, Bethe, Gamow, and Ulam. Many of the scientific contributions were "firsts" that had a profound influence on subsequent developments over a wide spectrum of scientific activities. These included the pion-proton phase shift analysis, the nonlinear coupled oscillators, the study of the genetic code, importance sampling, two-dimensional hydrodynamics, Monte Carlo calculation of nuclear cascades, universalities of iterative functions and "antierical" chess.

This story is about the MANIAC, but really it's about Nick Metropolis, who conceived it, built it, and saw how to use it for a wide variety of problems. The circumstances in postwar Los Alamos were special. The war had brought together in Los Alamos an exceptional group of scientists with whom Nick had established a close relationship. Von Neumann, Fermi, Bethe, Teller, Ulam, Feynman, Gamow, Turkevich, Richtmyer, among others, were drawn to Nick because they enjoyed working with him. He took a deep interest in *your* problem and worked hard on it with you, invariably making an essential contribution. Those who returned to Los Alamos after the war were drawn irresistibly to the MANIAC where they were made cordially welcome. They went away enriched by their new experience and rewarded by new scientific results central to their interest. This is a happy tale of how one of the first of the modern computers got its start and what it was able to do in its early years.

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FERMI'S BRUNSVIGA

Forty-five years ago, at the beginning of World War II, Nick was 25 years old. There weren't any computers as we now know them. We used slide rules and adding machines, hand-operated machines. Machines with an electric motor were the exception rather than the rule.

I remember, particularly, the hand operated machine that Fermi used with remarkable deftness. It was made by Brunsviga, a German firm from the town of the same name, famous as the place where Gauss was born. This machine had a crank that you rotated by hand. To multiply, the crank was rotated the number of times called for by each digit of the multiplier, shifting the carriage for each successive digit in turn. Fermi had one of these machines when he was working in Rome. He brought it along with him when he came to Columbia in 1939. He was using it when I started working with him on the chain reaction, soon after his arrival. If we were working together and I made a calculation using my slide rule, he started cranking his machine. By the time I announced my result he was waiting—and grinning. A little later, I acquired a Marchant. When it became clear that he could not keep up with me then, he gave up the Brunsviga and got a Marchant of his own. Fermi could never resist the opportunity to calculate faster!

It seemed to me that Fermi was always calculating something. It was Fermi's view that nature revealed itself through experiments devised to test it. You may construct a theory to explain what was going on, but unless the numbers come out right you can't be sure the theory is right. So you have to do a lot of calculations.

FERMI MONTE CARLO

I thought you might like to hear a little more about how Fermi used the Monte Carlo method long before it had the name. The story comes from Emilio Segrè, who told me that Fermi used the method in 1934 when he began making calculations on neutron diffusion.

Fermi liked to come to the lab in the morning and surprise his colleagues with a pronouncement of some new insight. To understand this better you should know that Fermi had insomnia and it got him up at 4 AM. That's when he'd work on theory and make calculations. For making quick calculations he had a whole bag of tricks, and the Monte Carlo method (a hand variety) was one of them.

He was in Rome. Joliet and Irène Curie had just discovered the artificial radioactivity produced by α -particle bombardment of light elements. This was in 1933. The neutron had been discovered just one year

before. These two facts gave Fermi the idea that neutrons, having no charge at all, would be much more effective than α particles in producing nuclear transformations. They would not be repelled by the Coulomb barrier and could, therefore, penetrate the nuclei of all atoms, whatever their charge.

It was an exciting idea. Fermi was working with a small but enthusiastic group; Amaldi, d'Augostino, Pontecorvo, Rasetti, and Segrè. Early in the course of the work some very peculiar effects were noted. The radioactivity obtained depended a whole lot on where the irradiation was carried out. In particular, the activity induced in silver was much greater when carried out on a wooden table than on one with a marble top. To clear up the mystery Fermi decided to try filtering the neutrons through various substances. His first idea was to use lead, but then at the last minute, for no apparent reason, he substituted paraffin instead. The effect on the activity of silver was phenomenal, and everyone went home deeply puzzled. By the next morning Fermi had figured what was happening and was ready with a most profound pronouncement. The neutrons were being slowed down by collisions with the nuclei of the filter. Among nuclei, hydrogen would be particularly effective in slowing down the neutrons. Fermi used Monte Carlo to get a better picture of what was going on. Later, in working out the detailed theory, Monte Carlo helped him choose a suitable functional form, Gaussian, exponential, or other, in analyzing the slowing down process.

It was a major discovery. Slow neutrons had very large cross-sections for nuclear reactions and produced a large class of artificially radioactive substances. This work won Fermi the Nobel Prize in 1938. It led directly to the chain reaction in Chicago in 1942, and to the establishment of Los Alamos in 1943.

Fermi never wrote up his use of the Monte Carlo method, but he told the story I've recited here to Emilio Segrè many years later when the method had its name and was coming into wide use. Mention is made in Segrè's introduction to the neutron papers in *The Collected Works of Enrico Fermi*.

By this time you may be wondering whether this is a talk about Nick Metropolis or about Enrico Fermi. Let me reassure you; it's about Nick Metropolis. It's just that there's a close connection, as will become more evident in due course.

MARCHANT REPAIR

It is 10 years later. The scene has shifted to Los Alamos. We find our hero busily repairing Marchant calculators. When Los Alamos was set up

in 1943 there were no calculators. There was, however, an obvious and urgent need to carry out a large variety of calculations. The Lab promptly purchased a large number of mechanical calculators and set up a hand-computing facility. The mechanical calculators were Friedens and Marchants. They were intensively used and soon began to show signs of wear and tear. Too many were out for repair, too few in useful service. It took too long to have the machines repaired by returning them to the manufacturer. To fill the breach, Nick, and Dick Feynman set up a shop for calculator repair. They traced the mechanical linkages to find the source of the jams and the slippage and learned how the machines worked. Soon they were diagnosing problems quickly and restoring the machines to service in short order. When the administration discovered this extracurricular activity there was some sharp criticism and the service was interrupted. But not for long. There was a strong demand for working machines and the service was soon reinstated.

PUNCHED-CARD MACHINES

In the fall of 1943 it became apparent that large computational problems were straining the capacity of the hand-computing facilities. Dana Mitchell suggested the use of IBM punched-card machines, and soon a complement of these arrived at the Lab. At this time Nick and Feynman set up a competition between the hand calculators and the punched-card machines. For the first two days the hand-computing group kept up with the punched cards, but after that the punched cards moved ahead. It became clear that the people operating the hand calculators tired and could not maintain their initial fast pace. The punched-card machines didn't tire and continued computing steadily.

As the atomic-bomb project entered its final phases in late 1944, the pressure for greater computation production increased. Nick became increasingly involved with punched-card operations. These continued, under Feynman's direction, at a frantic pace in the final months until the Japanese surrender, August 15, 1945, after which things became more leisurely.

ENIAC

The great step forward in computing was the introduction of electronics. This was done most successfully in the ENIAC (Electronic Numerical Integrator And Computer). This machine was designed and built by a group of engineers under the direction of Pres Eckert and John Mauchly. It was the first digital electronic general purpose scientific com-

puter. It used 18,000 vacuum tubes and computed 1000 times as fast as its electro-mechanical competitors. Interestingly enough, the ENIAC was built during wartime on the promise that it would calculate ballistic trajectories at least 10 times as fast as the mechanical differential analyzers then in use.

The connection between the ENIAC and Los Alamos was through Johnny von Neumann. He was a consultant at the Aberdeen Proving Grounds, the contractor for the ENIAC, and at Los Alamos. The brilliant von Neumann was very excited about the development of the ENIAC. He took a deep interest in its design and thought a good deal about what could be done with it. When, in early 1945, he told Nick, Edward Teller, and Stan Frankel about what the Eckert and Mauchly team was undertaking, they were enchanted by the prospects. A Los Alamos problem was proposed and Nick and Frankel were invited to put it to the ENIAC. They seized this great opportunity eagerly.

STORED PROGRAM

At this time the idea of the stored program had already been recognized and discussions were going forward on the next stage of the development, the EDVAC. The ENIAC was programmed by plugging cables and wires and setting switches, using a huge plugboard that was distributed over the entire machine. Figure 1 shows the ENIAC and the way

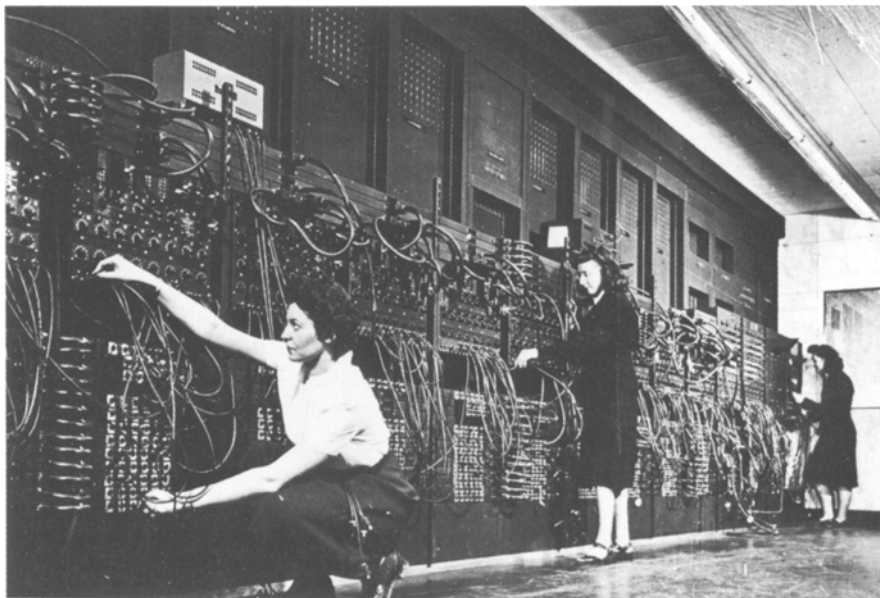


Fig. 1. Programming panels and cables of the Eniac. (Courtesy of N. Metropolis.)

the wiring between plugboards was done. This was an awkward and tedious way to tell the machine what to calculate and what to do with the result. This was the way the punched-card machines were handled. But the ENIAC functions were controlled by gates set by a two-digit code. On one of his many visits to Los Alamos (circa 1947), von Neumann described a suggestion of Richard Clippinger of the Ballistic Research Laboratory that the ENIAC might be converted into a limited stored program mode. The idea was to rearrange the so-called function tables, normally used to store 300 twelve-digit numbers set by manual switches, to store up to 1800 two-decimal digits, each pair corresponding to an instruction. These would be interrogated sequentially (including loops), making the transition from one problem to the next more efficient. Figure 2 shows the function table of the ENIAC.

This suggestion made a deep impression of Nick, and on one of his visits to the ENIAC in early 1948 he learned about the construction of a new panel to augment one of the logical operations. It was a one-input-hundred-output matrix and it occurred to Nick that this could be used to interpret the instruction pairs in the proposed control mode. When he mentioned this to von Neumann he was encouraged to proceed. The scheme was implemented on the ENIAC forthwith. Nick's set of problems—the first Monte Carlos—were run in the new mode.⁽¹⁾

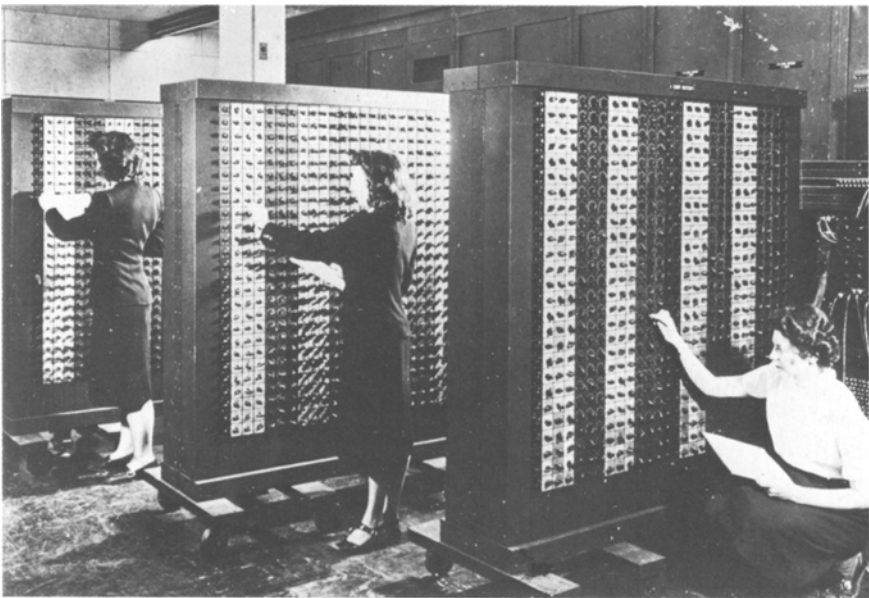


Fig. 2. Function tables of the Eniac. (Courtesy of N. Metropolis.)

THE MANIAC

After the war Nick and Stan Frankel joined the faculty of the University of Chicago. They expected to develop a major activity in computing. When this did not materialize as quickly as they had hoped they made other plans.

In 1948, Carson Mark, head of the Theoretical Division at Los Alamos, invited Nick to develop a research effort there. Nick was ready, willing, and eager, and the MANIAC (Mathematical Analyzer, Integrator And Computer) was born.

The MANIAC was designed according to von Neumann principles. These principles were set forth in a remarkable publication by A. W. Burks, H. H. Goldstine, and J. von Neumann.⁽²⁾ The MANIAC borrowed heavily from the IAS, the Institute for Advanced Study computer being built at Princeton under von Neumann's direction, since it came later it avoided many of the pitfalls that held up the prototype. The MANIAC was not the only machine to follow that course. There was the SEAC at the National Bureau of Standards, the ORACLE at Oak Ridge, the ILLIAC at the University of Illinois, the AVIDAC at Argonne, among others that participated in the outburst of activity following the end of World War II. The ENIAC started a revolution that continues unabated to this day, with no end yet in sight. The unusual success of the MANIAC was due primarily to the personality and motivation of Nick Metropolis, together with the group of highly capable engineers and programmers he assembled to help him build it and make it run. The original engineers were R. Merwin, H. Parsons, J. Richardson, H. Demuth, W. Orvedahl, and E. Klein. The importance of programming aids was recognized early. Led by J. Jackson, studies were begun on assembly languages and an assembler was produced at an early date. M. Wells et. al. launched the MADCAP development that led to a high-level language and compiler.

FERMI-METROPOLIS

With the MANIAC up and running Nick could hardly have gotten greater satisfaction than from the praise and interest shown by so many of the illustrious scientists with whom he had kept close contact from wartime Los Alamos days. The supreme accolade was from Enrico Fermi. For Fermi the MANIAC was just wonderful. He could hardly wait to get his hands on it. He loved to calculate, the faster the better, and here was his good friend Nick Metropolis with the fastest machine in the world, offering to introduce him to its mysteries and let him run it himself.

Fermi liked to come to Los Alamos during the summer months. He

had been in Los Alamos during the war and claimed that if Los Alamos could have become a university he wouldn't have left. His compromise was to spend the nine months of the academic year at the University of Chicago and most of the summer at Los Alamos.

When he came to Los Alamos in the summer of 1952 Fermi had a problem that was ideally suited to the MANIAC. He wanted to analyze, by the phase-shift method, the pion-proton scattering experiments he had been carrying on in Chicago with his collaborators at the new 450 MeV synchrocyclotron. My connection with this was analogous to Nick's except that in this case I had built the cyclotron. I also helped build the apparatus and carry out the measurements. The other collaborators were D. E. Nagle and E. A. Long, on the faculty, and my graduate students, R. L. Martin, G. Yodh, and M. Glicksman.⁽³⁾

The results had been as striking as they were surprising. The scattering appeared to have a resonant behavior. It was a major discovery, an excited state of the proton, a new particle now known as the Δ . A phase-shift analysis was called for to establish, in a definitive way, the quantum states involved and their amplitudes.

MASS TABLES

Fermi had a knack for coming up with problems whose computation matched the means available. Some years before, when the punched card machines were the principal means for computing, Fermi posed the problem of calculating a table of atomic masses using a semi-empirical mass formula he had devised on the von Weizsäcker model. Nick organized the calculation and the preparation of the tables. They turned out to be very useful and were widely used. I still have my copy.⁽⁴⁾

FERMI LECTURES

When Fermi returned to Chicago at the end of September he was bursting with all the new knowledge he had acquired while working with Nick. He announced that he would give a series of lectures on digital computing. We were treated to a magnificent course, with Fermi at his best. We learned, for the first time, about binary (and hexadecimal) arithmetic, Boolean algebra, and linear programming. With this kind of introduction we were easy converts to the cause of computers in science. The gospel according to Nick Metropolis was taking effect.

PROGRAMMING THE MANIAC

To show how closely Fermi interacted with the MANIAC, I'd like to show you some viewgraphs of some of his programming efforts, done in his own hand. Remember, these were the days before Fortran. Programming was done at the lowest level, in machine language. Figure 3 shows a sub-program Fermi wrote to convert the data in memory into decimals and printing the results. Figure 4 is a block diagram of the program for calculating the phase shifts by finding a minimum χ^2 in a fit to the data. Figure 5 is a printout of the program from the MANIAC. Note the use of hexadecimal numbers. The comments are written in Fermi's hand.

TELLER'S COLLOQUIUM

There's an amusing story about Edward Teller that fits in here. Remember that Nick had been a member of Teller's group when they were working on the thermonuclear bomb (the "super"). Not to be outdone by Fermi, he announced that he would give a colloquium on the subject of computers. When the notice appeared it didn't convey exactly the impression he had intended. It read:

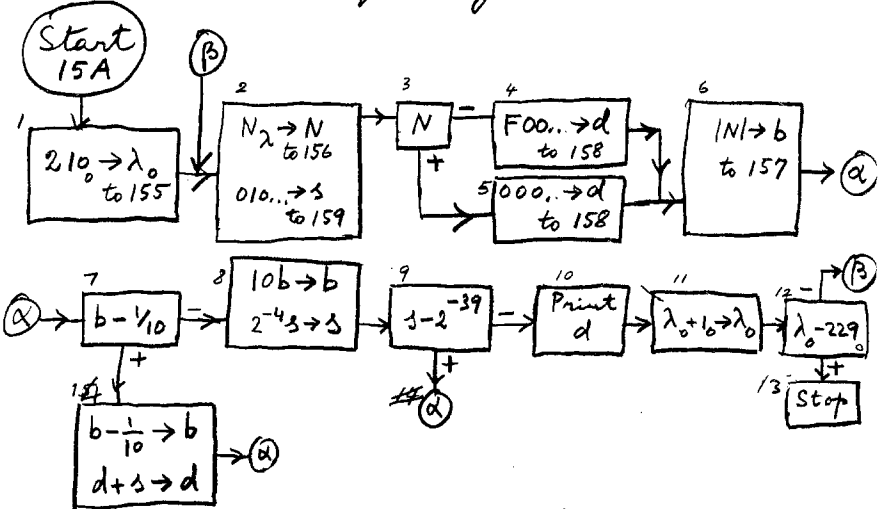
Edward Teller
The MANIAC

PHASE-SHIFT ANALYSIS

In this period, 1953–1954, the phase shift analysis problem occupied center stage of the elementary particle physics community. At the Rochester Conferences held in those and subsequent years you could talk about α_{33} and α_{31} and everyone would know that these were the phase shifts of the pion–proton scattering. The physics was important, the Δ was a new particle. There were also questions raised about nonlinear minimization problem that led to much research in applied mathematics. The existence of a multitude of solutions in the hyperspace of the parameters being dealt with was a new and suprising experience for us. The virtuosity of the computer almost made us lose sight of the discovery we had made. The computer found many sets of phase shifts that gave good fits to the data, leaving open the question whether the resonant solution was the correct one. The resonant solution was appealing in that it accounted for all of the unusual features found in the experiments, but the other nonresonant solutions were not easy to rule out.

Not satisfied with the way Fermi was handling the problem, and con-

— Flow diagram for converting memory 210-221 to decimals and printing results.



1	15A	m to AC	150	8	164	L	001	14	16E	m to A h	158
	AA	A to m	155		DE	A to m	14F		BA	A to m	158
2	15B	a to AC	010	9	165	L	002	15	16F	T	162
	EF	A to m	159		DE	m to A h	14F		CA		
3	15C	m to AC	155	10	166	A to m	157	16			
	AA	S to m	15D		DC	m to AC	159				
4	15D	m to AC	[lambda_0]	11	167	R	004	17			
	AA	A to m	156		EE	A to m	159				
5	15E	C	160	12	168	m to A h -	154	18			
	CC	a to AC	FOO		BB	C	162				
6	15F	A to m	158	13	169	Print	158	19			
	DC	T	161		EA	m to AC	151				
7	160	a to AC	000	14	16A	m to A h	155	20			
	EF	A to m	158		BA	A to m	155				
8	161	m to A h	156	15	16B	m to A h -	152	21			
	AE	A to m	157		BB	C	16C				
9	162	m to AC	157	16	16C	T	15B	22			
	AA	m to A h -	153		CA	Stop					
10	163	C	16D	17	16D	A to m	157	23			
	CC				DC		150				

Fig. 3. Fermi's program for the MANIAC: converting memory to decimal and printing the results.

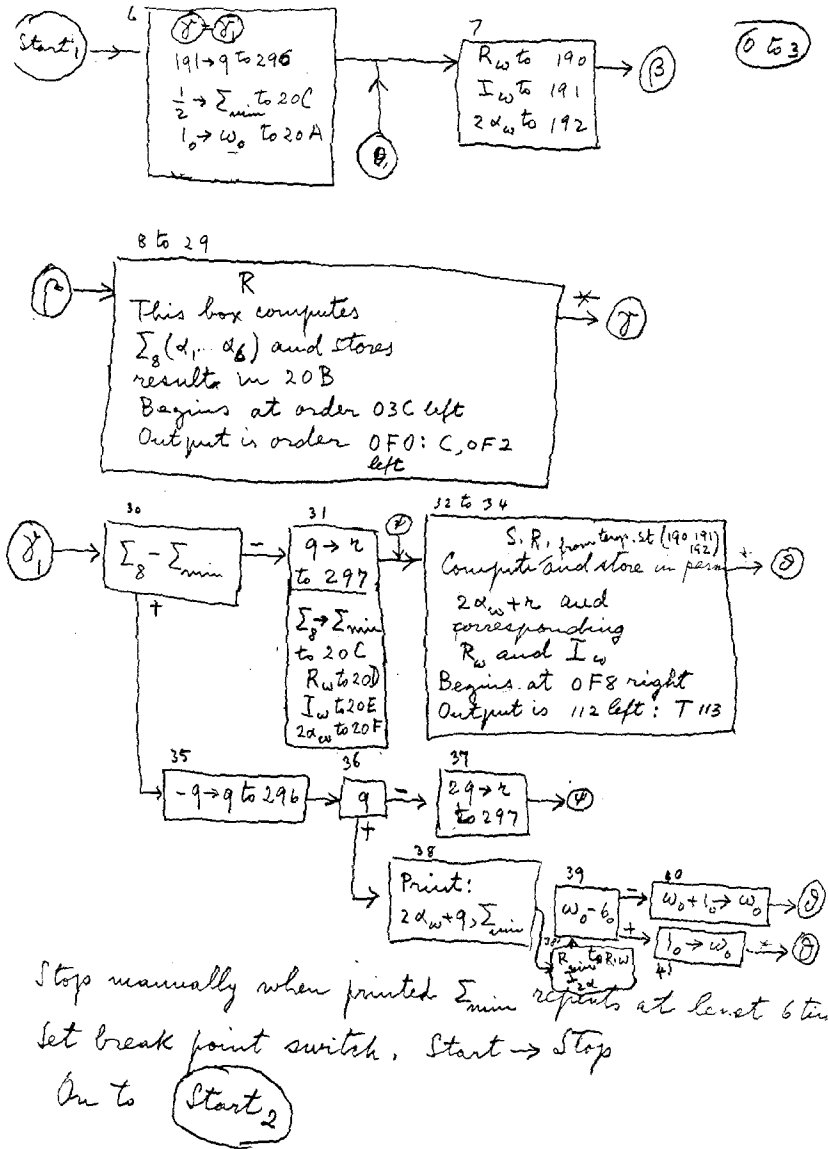


Fig. 4. Fermi's program for the MANIAC: calculation of the phase shifts.

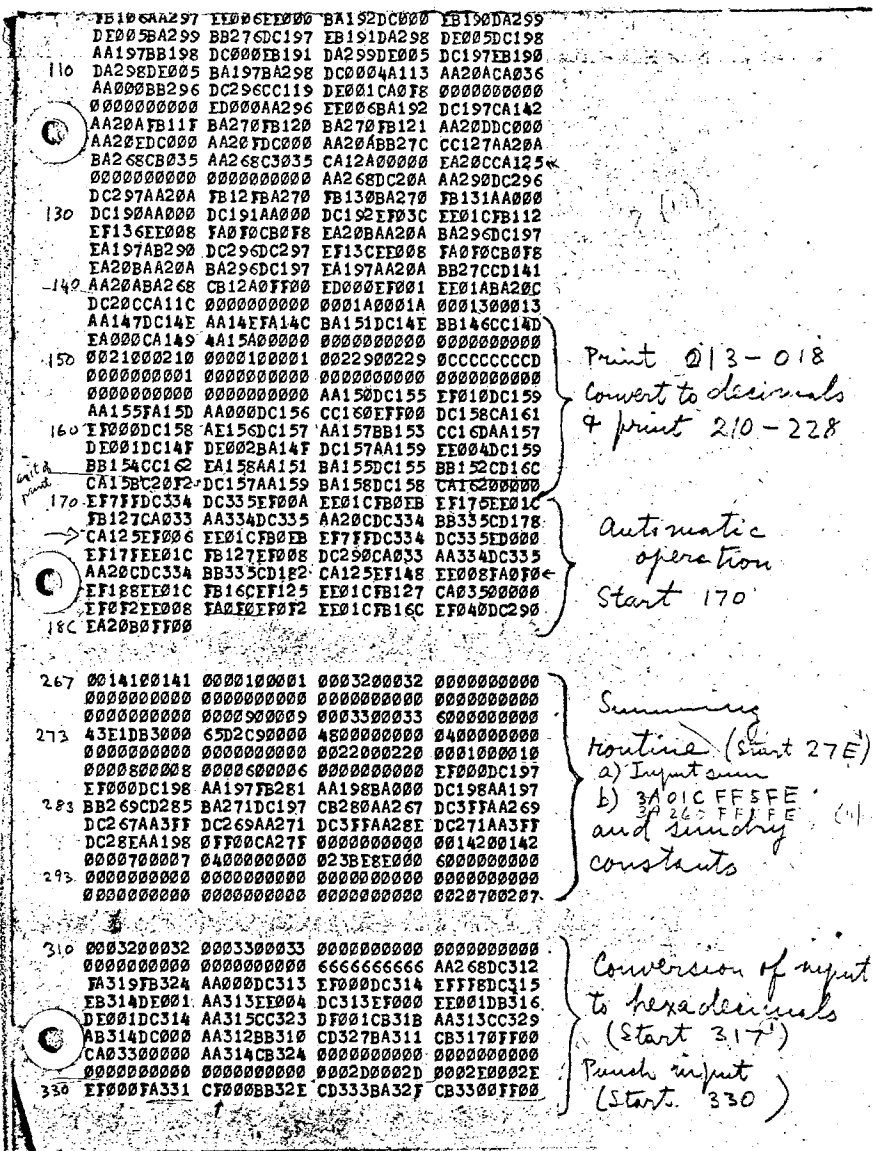


Fig. 5. Printout of Fermi's program for the MANIAC. The program is in machine language, written with hexadecimal numbers.

vinced that he could do better by including additional physics arguments, Hans Bethe made himself a great expert in phase-shift analysis. He enlisted Nick's aid and, with Fred de Hoffmann to help, mounted a competitive program. Nick was on both sides. In matters of science he didn't have favorites.^(5,6)

In the end, I think the problem was handled best by two of my graduate students, R. L. Martin and M. Glicksman, working separately. They didn't have a fancy computer but, using graphical methods with simplified but plausible assumptions, came up with the resonant solution that turned out to be the correct one. It was a lesson in the use of computers that should be a caution to us all.

NONLINEAR OSCILLATORS

I now turn to a number of applications of the MANIAC which were seminal in nature. They were examples of how the computer opened new possibilities for scientific investigation, sometimes with suprising results.

In the summer of 1953 Fermi raised the question of the nature of the approach to equilibrium of a vibrating nonlinear string initially in a single oscillatory mode. The idea was to have the MANIAC carry out the experiment. Together with Stan Ulam and John Pasta⁽⁷⁾ a test problem was set up and run. As expected, the computations showed that the initial vibrational energy gradually transferred into neighboring modes and eventually achieved equilibrium, the time taken being the so-called relaxation time.

But the completely unexpected happened one day when a typical problem was being computed. Owing to a very lively and distracting discussion, the computations continued beyond the usual cutoff. The results were so strange and mysterious that everyone around was quick to assume that the computer, the traditional whipping boy, had gone awry. The vibrational energy had returned to its initial state, within a few percent! The rest is history—nonlinear studies were shown to have some fascinating aspects. The ideas of soliton theory emerged and there was an outpouring of papers that became a minor industry. Today this classic work is known as the FPU problem.

GAMOW

The Metropolis circle of famous scientists included George Gamow. At the time Gamow was introduced to the MANIAC his interest had turned to biology. He already had the idea that the genetic code was vested in four nucleic acids which somehow were able to code for the 20 amino acids

from which proteins are made. Some of these early studies of the genetic code were carried out on the MANIAC.⁽⁸⁾

On the less serious side he was writing a book, *Mr. Tompkins Learns the Facts of Life*. Gamow was so fascinated by the MANIAC that in the book he introduces him to Mr. Tompkins. He wants to explain to Mr. Tompkins how the brain works but finds it more expedient to have MANIAC explain how he works. Figure 6 is an inimitable Gamow cartoon drawn to show how the computer works. This audience will appreciate how appealingly the essentials are presented.

IMPORTANCE SAMPLING

An important advance in the use of the Monte Carlo method came out of a collaboration with Edward Teller. Teller, with obvious delight at

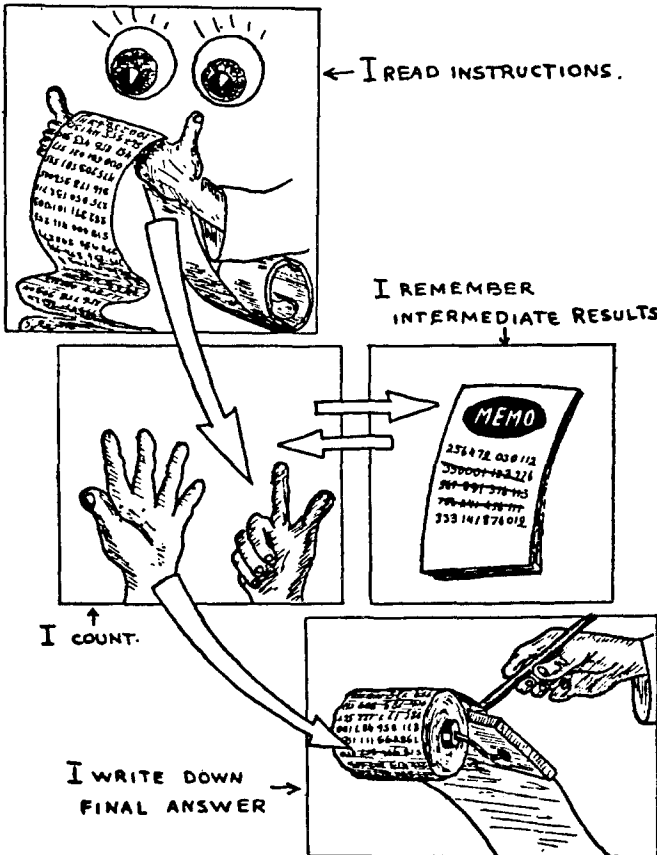


Fig. 6. Mr. Tompkins learns how the MANIAC works.

gaining access to such a marvelous toy proposed that it be used to carry out calculations on the equation of state in two dimensions for rigid spheres, using the Monte Carlo method. These calculations introduced the idea of what is now known as importance sampling, also referred to as the Metropolis algorithm.⁽⁹⁾ The scheme greatly improves the effectiveness of the Monte Carlo method. It is in widespread use today, as participants in this conference have made evident.

ITERATIVE FUNCTIONS

One of the surprising consequences of some early studies of iterative transformations by Metropolis, Stein, and Stein⁽¹⁰⁾ was the discovery of their universal properties. The work was an inspiration to Mitchell Feigenbaum who took it up a few years ago and used it to show how such functions lead to a theory of the onset of turbulence and chaotic behavior. The subject has turned out to be as important as it is exciting. It has fired the imagination of many who are intrigued by the curious aspects of non-linear behavior. It is currently being widely developed, a good example of computer-driven mathematics.



Fig. 7. Paul Stein and Nick Metropolis at the MANIAC and the 6×6 anticlerical chess board.

MANIAC EVOLUTION

Before continuing with the list of scientific triumphs of the MANIAC, let me make clear how the MANIAC evolved over the years. MANIAC II was a successor with many improvements started in Los Alamos in 1955. MANIAC III was developed at the University of Chicago when Nick returned there to head the newly formed Institute for Computer Research. It used the latest solid-state circuitry. Nick returned to Los Alamos in 1965, but by this time the computing needs of the Lab had increased dramatically and were being supplied by commercial machines. In this climate the Lab made the unfortunate decision to abandon the kind of original research that was special to the MANIAC project and the machine was turned off.

SCIENTIFIC TRIUMPHS

Figure 8 lists 10 of the many scientific uses of the MANIAC. These have been chosen to emphasize the distinction of the men who came to

SCIENTIFIC USES OF THE MANIAC

- 1) Fermi: The pion - proton phase shift analysis
- 2) Bethe - deHoffmann: Phase shift analysis
- 3) Fermi, Pasta, Ulam: Nonlinear coupled oscillators
- 4) Gamow: Genetic code
- 5) Teller: Equation of state: Importance sampling.
- 6) Von Neumann: Two-dimensional hydrodynamics
- 7) Stein and Stein: Universalities of iterative functions.
- 8) Turkevich: Nuclear cascades using Monte Carlo
- 9) M. Wells: Anti-clerical chess
- 10) Ulam: The lucky numbers

Metropolis a co-author except 3), 9).

Fig. 8. Scientific uses of the MANIAC.

work with Nick as well as the variety and importance of what they were able to accomplish. In addition to the papers I've discussed already, other noteworthy studies on the list include a classic study of nuclear cascades from the interactions of high-energy particles on heavy particles using Monte Carlo techniques, done with A. Turkevich.⁽¹¹⁾ J. von Neumann showed how to attack the two-dimensional flow of two incompressible fluids under gravitational and hydrodynamic forces.⁽¹²⁾ M. Wells et. al. prepared the first program to develop a strategy for "anticlerical" chess.⁽¹³⁾

MONTE CARLO METROPOLIS

Nick you'll remember those halcyon days
The Monte Carlo method was in its earliest phase.
You went to the ENIAC and got it to load
Problems you wanted in a stored program mode.
The Monte Carlos you gave it opened a crack
That helped you decide to build MANIAC.
MANIAC came out as a marvelous toy
A machine you could work with and really enjoy.
You called on your friends to join in the fun
And it wasn't too long before they'd begun.
There was Teller, Gamow, Turkevich too
Of others not mentioned there were quite a few
But these in particular knew that their goal
Was through Monte Carlo in the MANIAC's soul.
Those were seminal papers, seeds had been sown
That's how Monte Carlo came into its own.

Herb Anderson

9/6/85

Fig. 9. Poem: Monte Carlo Metropolis.

This was on a 6×6 board with bishops removed. It was a highly amusing experience, but in addition, many implications for subsequent games of strategy emerged. Figure 7 shows MANIAC I, with Paul Stein on the left, Nick Metropolis on the right, and the 6×6 board for “ant clerical” chess between them. Finally, I mention an amusing work by Stan Ulam and company introducing the notion of “lucky numbers,” a generalization of the ordinary prime numbers with many similar properties.⁽¹⁴⁾

POEM

I’ve always had the idea that when there is a birthday there ought to be a poem. Here, in Fig. 9, is my own modest offering. I’ve called it *Monte Carlo Metropolis*.

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